Communication of sab additive measures on Jordan banach algebra

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ABSTRACT

This article discusses the relationship between a subadditive measure and a trace on JBW – algebra.

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1. INTRODUCTION

It is known that continuation of a measure on idempotents of a JBW-algebra on the whole algebra is a trace and any state is connected with the trace using some positive element with the unit norm [1]. A similar fact for subadditive measures does not hold both on the Von Neumann algebra and in Jordan Banach algebras.

The article is devoted to finding good connections between a subadditive measure and a trace on JBW - algebras.

Empty A - JBW - algebra, ∇ - the logic of idempotents A.

Definition 1. A mapping $m : \nabla \to [0, \infty]$ is called a subadditive measure on ∇ if it satisfies the following conditions:

1) $m(\theta) = 0, \quad m(p) = 0 \Rightarrow p = \theta$

2) $p \le q \Rightarrow m(p) \le m(q)$

3) $p \sim q \Rightarrow m(p) = m(q)$

4) $m(p \vee q) \le m(p) + m(q)$

5) $p_n \uparrow p \Rightarrow m(p_n) \to m(q)$

Definition 2. A subadditive measure m is called finite if $m(1) < +\infty$;

Semifinite if for any idempotent $q \in \nabla, q \neq 0, q \le p$ there exists an idempotent $m(q) < +\infty$ such that.

Examples 1. The narrowing of the trace from A to ∇ is a subadditive measure.

2. Let τ be a trace on A, γ a continuous map of R to R with the properties: $\gamma(0) = 0, \ \gamma(x) \le \gamma(y)$ for $x \le y$ and $\gamma(x + y) \le \gamma(x) + \gamma(y)$ for all $x, y \in R$. Then $m(p) = \gamma(\tau(p))$ is a subadditive measure on.

Types of JBW - algebras are not related to the finiteness or semic finiteness of subadditive measures. For example, if a JBW - algebra is of type I_∞ with a semi-finite trace τ, then setting:

$$m(p) = \frac{\tau(p)}{1 + \tau(p)}, \quad \forall p \in \nabla,$$

We obtain a finite subadditive measure on ∇.

Definition 3. A mapping $l : A \to R$ is called a normal semi-additive functional if
1) l - positively;
2) $l(a) \le l(b)$ at $a \le b$, for all $a, b \in A^+$;
3) $l(a+b) \le l(a) + l(b)$. For all $a, b \in A^+$;
4) $a_n \uparrow a \Rightarrow l(a_n) \to l(a)$.

Examples 1. A trace is a normal semi-additive functional.

2. For any $x \in A$ we put $l(x) = \| x \|$. It is easy to see that $l(x)$ is a semi-additive functional. The restriction to ∇ has the form

$$m(p) = \begin{cases} 1, & if \quad p \ne 0 \\ 0, & if \quad p = 0 \end{cases}$$

This is a subadditive measure on ∇.

Theorem 4. Let l be a normal semi-additive functional on A. If $l(U_S a) = l(a)$, $a \in A$ for all, and s is symmetry in A, then the restriction of the semi-additive functional l from A to A is ∇ a subadditive measure.

Evidence. Obviously, $l(p) = l(q)$ for all $p, q \in \nabla$ at $p \sim q$. Let then $p \le q$. Then $q = (q - p) \nabla p$

$$l(q) = l((q - p)\nabla p) = l((q - p) + p) \le l(q - p) + l(p)$$

Means,

$$l(q) - l(p) \le l(q - p)$$

Further, since for JBW - algebras it is known that $p \nabla q - p \sim q - p \wedge q$, $l(p\nabla q - p) = l(q - p\wedge q)$ then. Therefore

$$l(p\nabla q) - l(p) \le l(p\nabla q - p) = l(q - p\wedge q) \le (q)$$

Consequently,

$$l(p\nabla q) \le l(p) + l(q)$$

The theorem is proved.

The question arises whether the converse is true, that is, is it possible to extend any subadditive measure to ∇ a semiadditive unitarily invariant functional on the whole algebra?

The following result was obtained in [2].

Let $A - JBW -$ algebra, ∇ - many idempotents in A.

Definition 5. [2] A function $\mu : \nabla \to [0, \infty]$ is called a positive measure if

$$\mu(e + f) = \mu(e) + \mu(f)$$

For anyone $e, f \in \nabla$ with the condition $ef = 0$.

A measure is called a probability measure if $\mu(1) = 1$.

A positive measure on ∇ is called a semiadditive measure if

$$\mu(e\nabla f) \le \mu(e) + \mu(f), \text{ for all } e, f \in \nabla.$$

Theorem 6. [2] Let A be a JBW algebra and let μ be a probabilistic semi-additive measure on ∇. Then μ uniquely extends to a trace state on A.

Therefore, for semi-additive measures, Gleason's theorem holds for all JBW - algebras (including JBW - algebras of type I_2).

In this work, the authors require that the additivity condition on orthogonal idempotents is satisfied for subadditive measures; we consider this problem in the general case.

If a trace is given in the JBW - algebra A, then by connecting the subadditive measure with the trace, the continuation problem can be solved.

Theorem 7. Let A - JBW be an algebra of type II_1, τ an exact, normal, finite trace and m be a subadditive measure on A. Then there exists a continuous function на on R^+ that is γ such that

$$m(p) = \gamma(\tau(p)) \text{ for anyone } p \in \nabla.$$

Theorem 8. If a JBW - algebra *A* is of type I_n, then any subadditive measure m can be expressed in terms of τ using some semiadditive function.

Note: From the construction of the semiadditive function γ in the previous arguments, it is clear that it is not unique. When constructing the values of γ between points S_1 and S_{i+1}, it suffices to take into account the fact that it satisfies the semi-additivity condition. In the simplest case, for example, for γ it is enough to replace the constant with a function between adjacent points [3].

From this reasoning it follows that every subadditive measure on ∇ the JBW - algebra A with a trace extends to a semiadditive unitarily invariant functional.

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